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Modification of: Error Log Analysis: Statistical Modeling and Heuristic Trend Analysis

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1. INTRODUCTION

Lin & Siewiorek [1], (henceforth LS) presented *heuristic* techniques for deciding if computer system problems could be traced to faulty hardware. LS also used "traditional statistical analysis methods" (their terminology) to distinguish between intermittent and transient errors. We show that these statistical techniques were "too traditional", *ie*, the "standard operating procedure" of fitting a distribution to data which were not identically distributed was adopted. This problem is discussed in detail in section 2; the reasons for the persistence of the misconceptions which have led to this prevailing, but inappropriate, approach are briefly discussed in section 3. Section 4 contains some brief observations concerning the LS heuristic approach.

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For any Cdf, $F_X(x) = \Pr\{X \le x\}$, the corresponding hazard function is:

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For $\alpha >> 1$, $z_X(x)$ increases very rapidly as x increases, and as $\alpha \to \infty$, $z_X(x)$ becomes an impulse at $1/\lambda$ [2: p 185]. If we keep sampling from this degenerate distribution, successive failures occur at $1/\lambda$, $2/\lambda$, $3/\lambda$, ...; *ie*, when the hazard function increases most rapidly, there is no tendency whatsoever for successive interarrival times to become smaller! For finite α, there is sampling variability which can result in a sequence of interarrival times which tend to get smaller — however, if we keep sampling from a Weibull distribution, with fixed parameters λ and (finite) α, sampling variability can also result in a sequence of interarrival times which tend to become larger, even if $\alpha > 1$. The basic point is that whenever we sample from a renewal process (*ie*, a nonterminating sequence of i.i.d. interarrival times) the successive interarrival times randomly vary around their common mean, which for the Weibull distribution is:

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A renewal process cannot be used to model a sequence of interarrival times that tend to decrease, because an increasing hazard function is a property of *one* interarrival time, rather than of a property of a *sequence of* interarrival times. Figure 1 portrays a sequence of interarrival times $X_1, X_2, X_3, \ldots$. The figure distinguishes between local time x, which is measured from the most recent error event, and global time t, which is measured from the origin for X_1, regardless of the number of error events. The rate of occurrence of failures of a sequence of interarrival times is:

$$v(t) \equiv d\,\mathrm{E}\{N(t)\}/dt,$$

Notation

$v(t)$ rate of occurrence of failures (rocof)
$N(t)$ observed number of failures in (0,t].

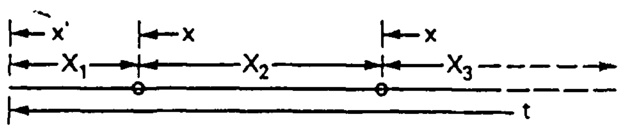

Figure 1. A Sequence of Interarrival Times

A necessary condition for modeling a decreasing sequence of interarrival times is to use a model for which $v(t)$ increases in t; this is not a sufficient condition [3: pp 41-42]. The nonhomogeneous Poisson process (NHPP) [3: pp 30-33] is presented as a suitable candidate by Thompson [4] and Rigdon & Basu [6], and in [3: pp 47-52]. Under the special case of a *power-law process* [3: p 101],

$$v(t) = \lambda\cdot\alpha\cdot(\lambda\cdot t)^{\alpha-1};$$

successive interarrival times tend to decrease for $\alpha > 1$, since the rate of occurrence of failures is increasing. As explained in section 3, in spite of superficial similarities between this NHPP and the Weibull distribution, there are crucial differences between these models.

3. REASONS FOR MISCONCEPTIONS

Ref 3 devotes 20 pages [3: pp 133-152] to tabulating & describing chronic misconceptions about repairable systems, and devotes 17 pages [3: pp 152-168] to reasons for these misconceptions. Instead of trying to summarize this material here, only the two chief causes of the widespread misconception (that a sequence of successively shorter *interarrival times* can be modeled by Weibull distributions with $\alpha > 1$) are presented.

1. The term "failure rate" is almost always defined as $h_X(x)$ but, as emphasized by Thompson [4], "failure rate" then is "naturally", and erroneously, interpreted as $v(t)$.[1] As explained in section 2, under the definition of "failure rate" as $h_X(x)$, there is no connection, in general, between increasing "failure rate" and a tendency for successive interarrival times to become shorter; unfortunately, under the incorrect interpretation of "failure rate" as:

$v(t) \equiv d\,\mathrm{E}\{N(t)\}/dt$, it appears that *increasing failure rate* corresponds to an *increasing number of failures per unit time* [5,6]. The almost universal use of "failure rate" for both $h_X(x)$ & $v(t)$ by practitioners & theorists is the chief cause of the lack of understanding that there are two different bathtub curves:

- $h_X(x)$ plotted against x for nonrepairable items
- $v(t)$ plotted against t for repairable items [7].

2a. The NHPP with $v(t) = \lambda\cdot\alpha\cdot(\lambda\cdot t)^{\alpha-1}$ has been referred to as a power-law process in these comments. This NHPP is widely and misleadingly/improperly known as a "Weibull process" in the literature. There is some connection between the so-called "Weibull process" and a Weibull distribution, *ie*, under the power-law process, time to *first failure* is Weibull distributed [3: pp 160-161]; but there are major differences between these models as well. There is just enough connection between the two models to make it especially important to emphasize the major distinctions between them [3,6]. These distinctions are blurred by the misleading/improper term, "Weibull process".

2b. Ref [1: section 3.4] used "Weibull process" in a very different sense, *viz*, a "Weibull process" is a renewal process with Weibull distributed interarrival times. Since this is another "natural" interpretation of "Weibull process", it provides another important reason for not using "Weibull process" as a synonym for *power law NHPP*. □

In addition to problems engendered by the terms, "failure rate" and "Weibull process", there are several subtleties encountered when distinguishing between the analysis of times to failure of nonrepairable items and the analysis of the interarrival times of a repairable system [3: pp 32-33, pp 51-52]. As emphasized in [8], reliability-oriented mathematical statisticians have almost ignored the discussion of these distinctions in their papers & books and have seldom even outlined appropriate techniques for repairable systems. As pointed out by Newton [9], for example, "... it is essential that sequencing is taken into account. It seems remarkable that so little attention has been given to this major potential pitfall. Ascher & Feingold [3] comment on the fact that among the hundreds of textbooks on reliability, only two (plus their own!) make any reference to the need to take sequencing into account." Moreover, even when repairable systems concepts & techniques are considered, the treatment is often very confusing: Basu & Rigdon [10] observed, "Much of what is written on repairable systems contains serious misconceptions and poor terminology. As Ascher & Feingold [3: p 133] state, '... the prevalent terminology could

[1]In fact, Thompson defines *failure rate* as $v(t)$ since, as he stresses, that is the way engineers interpret the term regardless of how it is defined. Ref [3] stresses that statisticians also often have fallen into this semantic trap, which they have set for engineers — and for themselves!

scarcely be more misleading if it had been designed to mislead- specifically, it has engendered such deep-seated misconceptions that it is extraordinarily difficult to supplant it with improved nomenclature'". It is not surprising, therefore, that statistical practitioners have been led astray by "traditional statistical analysis methods"!

4. COMMENTS ON THE DISPERSION FRAME TECHNIQUE

Ref [1: section 4] described the Dispersion Frame Technique (DFT) and illustrated its application to several types of computer hardware. The five heuristic DFT rules are designed to detect clusters of errors associated with a specific hardware problem against the background "noise" of randomly occurring transient errors. From the LS results, the DFT usually was successful in isolating such clusters, which associated with specific hardware problems. We have a few additional comments on mathematical analysis of the DFT and hope that they will stimulate further research.

1. The homogeneous Poisson process (HPP) is the model for maximum randomness of events occurring over time. As emphasized by/ inlar [11, p 80], however, the HPP appears (to the naive eye) to be clustered because the interarrival times are exponentially distributed; *ie*, there are many more short interarrival times than long ones. It would be useful, therefore, to use formal methods for distinguishing between the HPP and true clustering. Lewis [12, 13] provide techniques for distinguishing between an HPP and true clustering, and an NHPP *vs* an NHPP with additional clustering, respectively. Lewis [12] specifically applied his techniques to computer hardware problems; Jenkins [14] summarized the practical implications of those results.

2. Ref [1: section 4 (intro)] claimed, "These five [heuristic DFT] rules have been shown to mathematically cover a range of values for α, the Weibull shape parameter observed during the data analysis in section 3." LS did not provide such a mathematically oriented connection between the DFT rules and appropriate values of α, but this misconception should not have been put forth in the first place. That is (see section 2 above), LS inappropriately estimated the shape parameter of a Weibull distribution, rather than the shape parameter of a power law NHPP. If the NHPP is the appropriate model, then the estimate of the "Weibull distribution shape parameter" is meaningless. When interarrival times are not identically distributed, it is meaningless to estimate the nonexistent parameters of a nonexistent common distribution of interarrival times. Therefore, the conclusion stated at the end of [1: section 3.1] that, "$\alpha > 1$ is an oversimplification for intermittent faults" is unwarranted. Correspondingly, the conclusion reached near the end of [1: section 3.2] that, "simply looking for α greater that 1 is insufficient to identify the trend of an intermittent fault." is equally inappropriate. Moreover, all statements in the paper that are based on fitting Weibull distributions to interarrival times are questionable since the fitting of a distribution to data is appropriate only when there is no evidence that data are not identically distributed. □

In summary, the heuristic ground rules for the DFT established in [1: section 4] provide appropriate guidelines for distinguishing intermittent problems from transient "glitches". The results of [1: section 3] however, were based on "too traditional" statistical methods and will be revised in a future paper.

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Harold E. Ascher (M'61, SM'89) is a consultant and lecturer on reliability topics and is completing a book, *Statistical Analysis of Systems Reliability*. During 1992 Mar - May he was a Visiting Research Fellow in England and Scotland under a grant from the British Scientific and Engineering Research Council. Formerly, he was an Operations Research Analyst at the US Naval Research Laboratory where he participated in reliability programs on a wide variety of naval systems. Mr. Ascher received his BS in Physics from City College of New York in 1956 and his MS in Operations Research from New York University in 1970. He wrote, with Harry Feingold, the first book that extensively covers repairable systems, *Repairable Systems Reliability*. Mr. Ascher has written over 25 reliability oriented papers, and is a member of the American Statistical Association, IEEE, Society of Reliability Engineers, and the Washington Operations Research and Management Sciences Council.

(Continued on page 607)

$$\|\xi_n^* - \zeta^*\|_0^1 \le \sup_{0 \le b \le 1} \left| \int_0^\infty [\phi_n^*(bx) - \phi^*(bx)] dF(x) \right|$$

$$+ \sup_{0 \le b \le 1} \left| \int_0^\infty \left[\phi_n^*\left(\frac{x}{b}\right) - \phi^*\left(\frac{x}{b}\right) \right] dF(x) \right|$$

$$\le C_1 \|\phi_n^* - \phi^*\|_0^\theta + C_2 \epsilon$$

C_1 and C_2 are constants (independent of n).

Since $\|\phi_n^* - \phi^*\|_0^\theta \to 0$ a.s. as $n \to \infty$ and ϵ is arbitrary small, it follows that $\|\xi_n^* - \zeta^*\|_0^1 \to 0$ a.s. as $n \to \infty$. Hence, from [17: corollary 1, p 48] ζ_n converges weakly to ζ. The covariance kernel of ζ can be obtained by letting $\Psi(u) \equiv u$ in $\sigma^2(\Psi)$ in theorem 1. □

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